Anti-Aliasing Filters and Data Validity -The Inside Story

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With the world around us becoming more digital and less analog, the Test and Measurement industry is also embracing the new digital century. However, the transition is not always as smooth as we may like. For example, "*filter*," a familiar term, in principle performs the same function in both analog and digital applications but for a different reason. Analog systems can perform without a filter, but in the digital world you must have an analog filter. If you do not use a filter, it's like playing roulette at the risk of recording invalid data.

Let's start with analog instrumentation such as panel meters, chart recorders, tape recorders, oscilloscopes, etc. We'll discuss how a filter is used in these applications and why a filter in these instruments is not directly related to data accuracy.

Each of these analog instruments uses either mechanical or electronic means to measure an electrical quantity. Deflection of the needle in panel meters and chart recorders translates to a direct readout or ink on paper that is proportional to the speed of the electrical activity that is changing (frequency). Both of these recording devices are constructed mechanically and are limited in frequency response by mechanical constraints. Tape recorders and oscilloscopes are electronic and their frequency response is limited by either the magnetic head, the tape filament, or the ability to deflect an electronic beam. Signal frequency components that exceed the mechanical or electronic response capability of the measurement instrument are automatically eliminated. A filter used in these instruments smoothes out the readings or removes higher frequencies that are not of interest. These filters are not used to ensure data validity. In contrast, when using a digital sampling instrument you must have a low pass analog filter to avoid gambling with the quality of your data.

What is the difference between analog and digital systems that causes this problem? Fundamentally, the big difference between analog and digital is that the digital sampling system can only look at data at one moment in time. Many data points must be gathered to approximate what the real world looks like. If you take data at fixed time intervals, you can get a very distorted view of the world based on how often you sample the system.

For instance, if you sample a 50 Hz sine wave once every 20 msec (a sample period of 50 Hz), you will conclude that it's a DC source that provides constant voltage output. As a matter of fact, any multiple of 20 msec (40 msec, 60 msec, etc.) would give you the same result. But, if you monitor every 25 msec, you will see two different voltage levels and conclude that the circuit provides a 20 Hz square wave output. Both scenarios are shown in Figure 1. As you sample faster and faster, you will see a smooth 50 Hz sine wave. But what about the measurements you made at slower sampling rates? They are correct measurements at that specific moment in time but because the waveform's sampled interval is equal to or slower than the actual period of the wave-form, the data are not a true representation of the waveform. This is what we call **aliasing**, which **only occurs in digital sampling systems**.

How do we avoid this problem? One approach is to sample data at very high speeds. But, sampling at high speeds requires more storage space. The additional data also complicate our analysis because of the larger data files

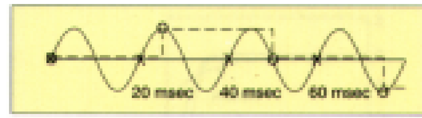

Figure 1. The solid line is the input waveform, a 50 Hz sine wave. The straight line describes the waveform if data points are taken 20 msec apart (labeled X). The dashed line describes the waveform if data points are taken 25 msec apart (labeled O).

we must deal with. The best alternative is to apply a low pass analog filter to limit the frequency response of the system. By using a sampling rate that matches the filter frequency, only waveforms that pass the filter are digitized, under-sampling is avoided, and only valid data are recorded.

The relationship between sampling rates and filter cutoff frequencies has been well studied. A simple equation determines this relationship expressed as "Sampling Ratio." *The Sampling Ratio (SR) is the minimum sample rate required over the maximum frequency of interest (the filter corner frequency) to accurately represent the waveform.*

$$SR = S/F_D = 1 + F_R/F_D$$

where S = sample rate, F_D = maximum desired data frequency, and F_R = frequency at the required attenuation. F_D specifies the corner frequency of a filter when it reaches attenuation at the -3 dB point. F_R is related to the attenuation required to maintain a desired precision of the data.[1] For a 0.01% (12 bit) system it is -80 dB, and for a 0.0015% (16 bit) system it is -96 dB. It is very easy to use the equation and apply it to any filter curve. It also confirms Nyquist's theorem: when you have a brick wall filter (i.e., the frequency at cutoff is

[1] Information about the Sampling Ratio equation is available in class notes from: the Tustin Technical Institute short course on Digital Data Acquisition by Strether Smith. For course information visit: http://www.ttiedu.com.

equal to the frequency at the desired attenuation), the sampling ratio will equal 2.

Table 1 shows some of the Sampling Ratios for typical filters currently used. Figure 2 shows the relationship between 2, 4, 6 and 8 pole filters.

Table 1. Sampling ratios for typical filters.

	–60 dB (0.1%)	–80 dB (0.01%)	–96 dB (0.0015%)
2 pole Bessel	41	128	321
2 pole Butterworth	33	101	252
4 pole Bessel	10	16	26
4 pole Butterworth	6.6	11	17
6 pole Bessel	6.5	9	12
6 pole Butterworth	4	5.6	7
8 pole Bessel	5.5	7	8.5
8 pole Butterworth	3.4	4.2	5
8 pole (Ellip., Cauer)	2.7	3	3.5
Linear Phase (R. C. Electronics)	2.4	2.45	2.5

There are other ways to avoid aliasing problems that may or may not work in a digital world. Some common techniques include:

Digital Filter. A digital filter will only work if the data you originally digitized are not aliased. You must use an analog filter to limit the frequency of incoming analog data before you digitize the data.

Delta-Sigma Converter. This is an A/D converter with a built-in digital filter that initially seems not to require an analog filter. Delta-Sigma is a sampling converter; it will alias. It uses a very high sampling rate internally, thus reducing the requirement for a sharp rolloff filter. But, it does require an analog filter. If you plan to change the sampling rate of the Delta-Sigma converter, you will also need a programmable analog filter to properly digitize the signal at the given sample rate.

Single Fixed Analog Filter. If you use a single fixed frequency analog filter, you can only have a fixed sampling rate data recording system. For instance, if you use a 6 pole Bessel filter at 20 kHz with a 16 bit system, you must have a sampling rate of 240 ksamples/sec (kSPS) (20 kHz x 12) to avoid aliasing (refer to Table 1). Refer to Figure 3 for a comparison of different anti-aliasing filter types. In addition, you cannot change your sampling rate to a lower frequency. A single fixed high frequency analog filter will not protect you at lower sampling rates.

Analog/Digital Filter Combination. A fixed analog filter and a digital filter with decimation can be combined to make a multi-sampling rate system work. The first stage analog filter and the digital sampling rate must be properly matched to avoid aliasing. Once non-aliased data are digitized, you must use a digital filter before decimating the data to a lower sampling rate, to avoid aliasing at this step. *You cannot cut corners with the digital filter.* If you decide to do engineering unit conversion or triggering as part of your digital calculation and do not use a sharp digital filter before decimation, the data will alias as it does with improper usage of analog filters in a digital sampling system. The R.C. Electronics Inc. DATAMAX II™ Instrumentation Recorder is an example of using analog/digital filters very efficiently to achieve a sampling ratio of 2.5 at 16 bit (-96 dB) attenuation to record data having a 40 Hz to 90 kHz analog bandwidth (see Figure 3).

Is a filter necessary if you know the frequency content of your waveform? The answer is no, you will not need a filter. The problem in the real world is that you are never sure what frequencies are on your signal line. Some of the high frequency noise sources that affect your data include switching power supplies, monitors, RF noise from your computer, AC power sources, etc. If you do not use the right filter, it is like playing roulette; you may collect good data and you may collect bad data, you never can tell. The only way to guarantee valid data is to use an anti-aliasing filter.

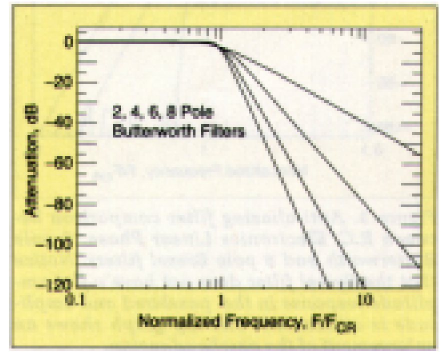

Figure 2. This graph shows why anti-aliasing filters are typically defined as 6 pole and above. With fewer than 6 poles, you have to sample much higher to avoid aliasing.

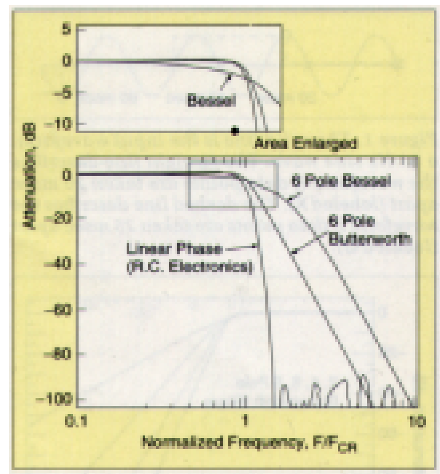

Figure 3. Anti-aliasing filter comparison between R.C. Electronics Linear Phase, 6 pole Butterworth, and 6 pole Bessel filters. Notice that the Bessel filter does not have a flat amplitude response in the passband and amplitude is attenuated. The top graph shows an enlargement of the passband region.